Topic 2 -<br>Linear first order ODES

A linear first order ODE is an  
\nequation of the form  
\n
$$
a_1(x)y' + a_0(x)y = g(x)
$$
  
\nIf we are considering an interval  $\Gamma$  where  
\n $a_1(x) \neq 0$  for any x in  $\Gamma$  then we  
\n $a_1(x) \neq 0$  for any x in  $\Gamma$  then we  
\n $a_1(x) \neq 0$  for any y in  $\Gamma$  then we  
\n $g' + a(x)y = b(x)$   
\n $y' + a(x)y = \frac{a_0(x)}{a_1(x)}$  and  $b(x) = \frac{g(x)}{a_1(x)}$ .  
\nWhere  $a(x) = \frac{a_0(x)}{a_1(x)}$  and  $b(x) = \frac{g(x)}{a_1(x)}$ .  
\nThis is the type of equation that  
\nwe will consider the now.

Suppose we have <sup>a</sup> linear first order ODE of the form  $\left|\begin{array}{cc}y' + a(x)y = b(x) & (\ast) & \rightarrow \end{array}\right|$ where a(x) and b(x) are  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ continuous on an open this Let's solve Suppose  $\phi(x)$  $(\ast)$  on  $\pm$ .  $\begin{array}{c} \n\searrow$  $y' + \frac{2xy}{a(x) = 2x} \xrightarrow{6(x) = x} 6(x) = x$ <br>  $y' + \frac{2xy}{a(x) = 2x} \xrightarrow{6(x) = x} 6(x) = x$ That is , suppose we have a<br>first order ODE of<br> $y' + a(x)y = b(x)$ <br>there  $a(x)$  and  $b(x)$ <br>et's solve this.<br>et's solve this.<br>(et's solve this.<br>(k) on I.<br>at is all x in I.<br>(all x in I.<br>(all x in I.<br>ivatur of alx) on I.  $\frac{15}{10(x)}\frac{6(x)}{6(x)} = 6(x)(x)$  $\frac{Ex:}{2xy = x}$   $2xy = x$   $x = (x) = x$   $x = (x) = x$   $x = x$   $a(x) = 2x$ <u>،</u> That is,<br> $\frac{f(x) + a(x) \phi(x)}{f(x) + a(x) \phi(x)}$  $A(x) = x$ derivative (\*) on 1.<br>
That is<br>  $\frac{f(x) + a(x) \phi(x) = b(x)}{b'(x) + a(x) \phi(x) - b(x)}$ <br>
for all x in I.<br>
Let A(x) be an unti-<br>
Let A(x) be an unti-<br>
Let A(x) be an unti-<br>
Let A(x) be an unti-Suppose we have<br>first order ODE<br> $y' + a(x)y =$ <br>where  $a(x)$  and<br>continuous on an<br>Let's solve this<br>Suppose  $\phi(x)$  solv<br>(k) on I.<br>That is<br>That is<br> $\phi'(x) + a(x) \phi(x) =$ <br>for all x in I.<br>Let  $A(x)$  be an<br>Let  $A(x)$  be an<br>Let  $A(x)$  be an<br>Mo Note: A(x) exists by the FTOC<br>Since a(x) is continuous.  $(x)$  be an arrived  $f(x)$  be an arrived  $f(x)$  be an  $f(x)$  on  $f(x)$ <br>A(x) exists by the FTOC<br>a(x) is continuous.

 $A(x)$  to get: Multiply (#\*) by c'  $A(x)$   $b(x)$  $e^{A(x)}\phi'(x) + a(x)\phi(x) = e$ This gives (by the product rule  $(fg)^2 = f'g + g't$ )  $\begin{bmatrix} 1 & 0 \\ 0 & \end{bmatrix}$   $\begin{bmatrix} 1 & 0 \\ 0 & \end{bmatrix}$ (e  $A(x)$   $\left( x \right)$  = e integrate Let  $B(x)$  be an anti-deriva side oleniuative<br>Figd pointed<br>I .<br>I . of  $e^{A(x)}b(x)$  on  $f$  . Let  $f$ if rege and only if tive<br>
It<br>
1 both Then  $\phi$  solves  $(\star)$  on  $\pm$  if  $\begin{array}{ccc} \vdots & \vdots & \vdots \end{array}$  $\frac{1}{x}$  $e^{A(x)}\phi(x) = B(x) + C$ where C is some consign constant.  $\zeta$  o  $\zeta$ <sup>4</sup> selves (\*) On ) 1<br>一<br>工 and only if  $- A(x)$   $A(x)$  $+$   $C\overline{e}$  $\phi(x) = B(x)e$ 

Since all the steps ubove are reversable since  $e^{A(x)} \neq 0$ We know we have found the general solution to (\*).



$$
\frac{E\times z}{\sqrt{2\pi}} \int_{0}^{1} 1/e^{x} dx = \int_{0}^{1} 1/e^{x} dx
$$
\n
$$
\frac{1}{2} \int_{0}^{1} 1(e^{x}) dx = \int_{0}^{1} 1
$$

$$
2x^{2}y'(x) + 2xe^{x^{2}}y(x) = xe^{x^{2}}
$$

Step 2: Undo the product rule on the left-hand side:  $(e^{x}y(x))' = xe^{x^2}$ 

sides to get: Step 3: Integrate both

 $e^{x} - y(x) = \frac{1}{2}e^{x} + C$  $xe^{2}dx = \frac{1}{2}\int e^{u}du$  $u = x^{2}$ <br> $u = 2xdx$  =  $\frac{1}{2}e^{2} + c$ <br> $\frac{1}{2}du = xdx$  =  $\frac{1}{2}e^{2} + c$ 

Step 4:  $y = \frac{1}{2} + C e^{-x}$ Thus, Constant C  $for some$ 



$$
\frac{Ex: Let's solve}{s_0(x) y} = \frac{sin(x)cos(x)}{b(x)cos(x)}
$$
\n
$$
y' + \frac{cos(x) y}{a(x)cos(x)} = \frac{sin(x)cos(x)}{b(x)cos(x)}
$$
\n
$$
\frac{1}{a(x)cos(x)} = \frac{cos(x)}{b(x)} = \frac{cos(x)}{x}
$$
\n
$$
\frac{3+e^{2} + 1}{2} = \frac{cos(x)}{x} = \frac{sin(x)}{x}
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$$
\frac{1}{2} = \frac{cos(x)}{x} = \frac{sin(x)}{x}
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\frac{sin(x)}{x} = \frac{sin(x)}{x}
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\n
$$
\frac{1}{2} = \frac{sin(x)}{x}
$$

Step 3: Integrate both sides.  $e^{\sin(x)}y(x) = \sin(x)e^{\sin(x)}e^{\sin(x)} + C$ 4 Sin(x) cus(x) esin(x) dx  $=\int t e^{t} dt = \pm e^{t} - \int e^{t} dt$  $\begin{array}{lll}\n\hline\n\text{L} &= \sin(x) \\
\text{d}t &= \cos(x) \, dx \\
\hline\n\text{L} &= \frac{1}{2} \int \frac{dx}{dx} = \frac{1}{2} \int \frac{dx}{dx} = \frac{1}{2} \int \frac{dx}{dx} \\
\hline\n\text{L} &= \frac{1}{2} \int \frac{dx}{dx} = \frac{1}{2} \int \frac{dx}{dx} = \frac{1}{2} \int \frac{dx}{dx} \\
\hline\n\text{L} &= \frac{1}{2} \int \frac{dx}{dx} = \frac{1}{2} \int \frac{dx}{dx} = \frac{1}{2} \int \frac{dx}{dx$  $=te^{t}-e^{t}+C$  $= sin(x)e^{sin(x)} - e^{sin(x)} + C$ Step 4: Thus,  $-sin(x)$  $= sin(x) - 1 + Ce$ C is some constant. Where

Ex: Consider the equation  $xy' + y = 3x^{3} + 1$  $X y'$  $=$   $(0, \infty)$  .  $J = (0, \infty)$ <br>Since  $x \neq 0$  on  $T$  we can divide by  $x$ to get  $3x^{2}+\frac{1}{x}$  $y' + \frac{1}{x}y =$  $x \neq 0$  un  $T$  We<br>  $\frac{1}{x}y = 3x^{2} + \frac{1}{x}$ <br>  $\frac{1}{\omega(x)}$   $\omega(x)$ Step 1: Let  $A(x) = ln(x)$ .  $\frac{4}{a(x)} + \frac{x}{a(x)}$ <br>Step 1: Let  $A(x) = ln(x)$ .<br>Then,  $A'(x) = \frac{1}{x}$  for all x in F. Then,  $A(x) = x$ <br> $A(x) = e^{\ln(x)} = x$  to get Multiply by <sup>e</sup>  $xy' + y = 3x^{3} + 1$ Step 2: Undo the product rule to get  $(xy)' = 3x^{3} + 1$ 

Step 3: Integrate both sides to yet  
\n
$$
xy = \int (3x^{3}+1) dx = \frac{3}{4}x^{4}+x + C
$$
\nStep 4: Thus,  
\n
$$
y = \frac{3}{4}x^{3}+1+\frac{c}{x}
$$
\nwhere C is a constant.

$$
\frac{S+ep4:}{}^{n} \text{ Thus,}
$$
\n
$$
y = \frac{3}{4} \times \frac{3}{4}l + \frac{c}{x}
$$
\nwhere C is a constant

$$
E \times : Solve
$$
\n
$$
x y' + y = 3 x^{3} + 1
$$
\n
$$
y(1) = 2
$$
\n
$$
00 \quad T = (0,100)
$$
\n
$$
F(s) = above
$$
\n
$$
x = x^{3} + 1 + 2x
$$
\n
$$
y = \frac{3}{4}x^{3} + 1 + 2x
$$
\n
$$
y = \frac{3}{4}x^{3} + 1 + 2x
$$
\n
$$
y = \frac{3}{4}(1)^{3} + 1 + 1
$$

$$
\begin{aligned}\nS_{01} \\
Z &= \frac{7}{4} + C\n\end{aligned}
$$

Thus,  

$$
c=\frac{L}{4}
$$

Thus,  

$$
y = \frac{3}{4}x^3 + 1 + \frac{c}{x}
$$
.